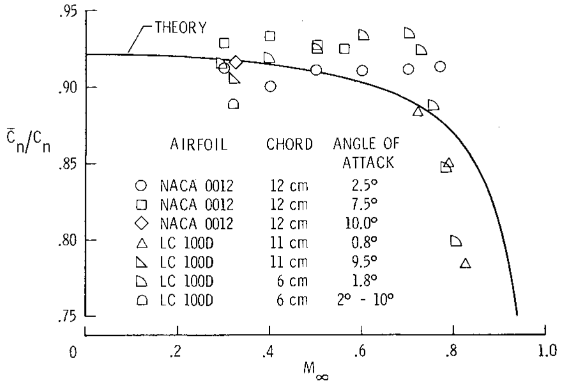

Fig. 2 Comparison of theory with data obtained in ONERA R1Ch wind tunnel with no sidewall suction upstream of model.

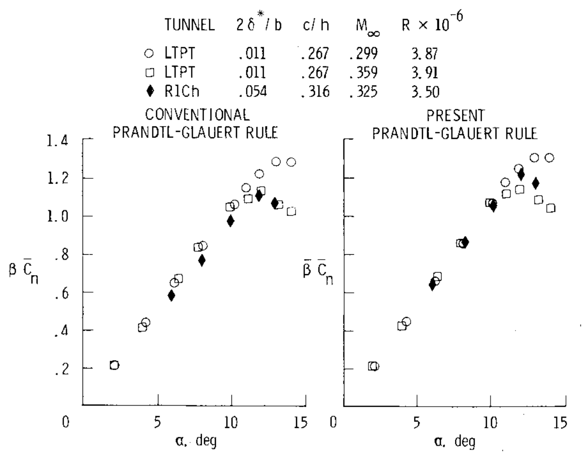

Fig. 3 Comparison of conventional and present form of Prandtl-Glauert rule for NACA 0012 airfoil.

additional data for the NACA 0012 airfoil and both models of the LC 100D airfoil are presented in Ref. 3. In Fig. 2, the experimental results for the normal-force coefficient ratio $\bar{C}_n/C_n$ obtained from Ref. 2 and a representative sample of the results obtained from Ref. 3 are compared with theoretical results obtained from Eq. (9). The subsonic results indicate good quantitative agreement as the incompressible limit is approached. The data for the supercritical airfoil show the same rapid decrease in $\bar{C}_n/C_n$ predicted by the theory as the freestream Mach number M_∞ increases toward one. The theoretical results for $\bar{C}_n/C_n$ for Mach numbers near the critical value differ from the experimental solution for both airfoils. This difference probably occurs because the theoretical solution for $\bar{C}_n/C_n$ does not account for nonlinear transonic effects.

For values of M_∞ greater than about 0.84 the experimental values for $\bar{C}_n/C_n$ for the supercritical LC 100D airfoil are greater than one. The experimental values for $\bar{C}_n/C_n$ for the NACA 0012 airfoil exceed one for values of M_∞ just larger than the critical value, and hence much smaller than the value for the supercritical airfoil. In fact, the effect occurs for the NACA 0012 airfoil before the beginning of the decrease in $\bar{C}_n/C_n$ caused by the singularity at $M_\infty = 1$. None of the experimental values for $\bar{C}_n/C_n$ greater than one are shown in Fig. 2. The effect is undoubtedly due to the presence of shock waves on the airfoils and the interaction for these shock waves with the sidewall boundary layers. The reason the LC 100D airfoil retains subsonic characteristics to higher freestream Mach numbers than the NACA 0012 airfoil is probably due to its supercritical design.

It is shown in Fig. 3 that the present theory can be used to correlate results for the variation of the normal-force coefficient with angle of attack obtained in wind tunnels with different values of the sidewall boundary-layer parameter. The airfoil used is the NACA 0012. The wind tunnels in which the data were obtained are the NASA Low-Turbulence Pressure Tunnel, which has closed walls, and the ONERA R1Ch tunnel with closed upper and lower walls. The displacement thickness parameter for the low-turbulence pressure tunnel is $2\delta^*/b = 0.011$. The data from this tunnel, which have not been published previously, were obtained by C. L. Ladson. The R1Ch data shown in Fig. 3 were obtained with no sidewall suction. It can be seen that the traditional function $\beta \bar{C}_n$ does not correlate the data between the two wind tunnels, but that the function $\bar{\beta}\bar{C}_n$ does.

The results presented in Fig. 3 are not corrected for interference from the closed upper and lower walls because the corrections for the two model-and-tunnel combinations used are almost the same. The analysis of Ref. 1 shows that the difference in the uncorrected normal-force coefficients for the two experiments due to interference from the upper and lower walls is only about 0.6% of the coefficient value.

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Improved Orthogonality Check for Measured Modes

80006

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Conventional Method

MANY structural dynamicists reply on an orthogonality check as a measure of the validity of normal modes derived from ground vibration testing. In order to perform this check, a mass model of the structure is used which has degrees of freedom that correspond to the points at which the modes are measured. This model is conventionally obtained by performing an approximate coordinate reduction, such as that of Guyan,[1] on a larger analytical model. Because this reduction should contain frequency dependent effects,[2,3] it is possible that it may give misleading results, especially when higher frequency modes are checked for orthogonality.

Improved Method

Consider the partitioned mass and stiffness matrices for a linear, undamped representation of a structure, where the upper left-hand coordinates represent the test points.

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$$K=\begin{bmatrix} K_1 & K_2 \\ K_2^T & K_4 \end{bmatrix} \quad M=\begin{bmatrix} M_1 & M_2 \\ M_2^T & M_4 \end{bmatrix}$$

The ith modal vector is also partitioned so that the upper portion ϕ_{1_i} contains the measured modal displacements and the lower portion ϕ_{2_i} is unknown

$$\phi_i=\begin{Bmatrix} \phi_{1_i} \\ \phi_{2_i} \end{Bmatrix}$$

The eigenvalue equation (for the ith mode)

$$(K-\omega_i^2 M)\phi_i=0 \tag{1}$$

may be written in partitioned form

$$\begin{bmatrix} K_1-\omega_i^2 M_1 & K_2-\omega_i^2 M_2 \\ K_2^T-\omega_i^2 M_2^T & K_4-\omega_i^2 M_4 \end{bmatrix} \begin{Bmatrix} \phi_{1_i} \\ \phi_{2_i} \end{Bmatrix}=0 \tag{2}$$

where ω_i is the measured natural frequency of the ith mode. From Eq. (2), the relationship between the measured portion of the mode and the unknown portion is (as in Ref. 3)

$$(K_4-\omega_i^2 M_4)\phi_{2_i}=-(K_2^T-\omega_i^2 M_2^T)\phi_{1_i} \tag{3}$$

or

$$\phi_i=T_i\phi_{1_i} \tag{4}$$

where

$$T_i=\begin{bmatrix} I \\ -(K_4-\omega_i^2 M_4)^{-1}(K_2^T-\omega_i^2 M_2^T) \end{bmatrix} \tag{5}$$

Equation (1) may be written using the full matrices and the measured modal displacements

$$(K-\omega_i^2 M)T_i\phi_{1_i}=0 \tag{6}$$

and the orthogonality relationship $\phi_i^T M\phi_i$ becomes

$$\phi_{1_i}^T T_i^T M T_j^T \phi_{1_j}=\delta_{ij} \tag{7}$$

where the modes are normalized to make

$$\phi_i^T M\phi_i=1$$

Note that when the effects of ω_i in Eq. (5) are small enough to be ignored, the expression $T_i^T M T_j$ in Eq. (7) is the reduced mass matrix of Ref. 1. When these effects must be included, $T_i^T M T_j$ may be considered to be a reduced mass matrix, but it will have different values for each pair of modes and, interestingly, will be unsymmetrical when $i \neq j$. (For intermediate values, i.e., ω small but not negligible, the approximation of Ref. 3 may be used.)

In the implementation of this procedure, it is suggested that the full modes be formed by solving Eq. (3) as a set of simultaneous equations and then using the full mass matrix for the orthogonality check.

Several additional comments are noteworthy:

1) This process may be considered as expensive on an absolute basis. However, when compared to the cost of the test and the value of the information gained, the additional costs must be rated as trivial.

2) The influence of frequency on orthogonality becomes more important for the measured modes with higher frequencies and probably will be quite large when modes in the acoustic range are being investigated.

3) When this method is not used, one may anticipate larger errors in the orthogonality check as the frequencies increase, even when the measured modes are exactly consistent with the analytical model.

4) The normalization of the modes by the suggested method inherently includes the effects of significant displacements which were not measured.

5) This process may allow the use of fewer measurement points than would be otherwise necessary.

6) A numerical illustration of this process may be found in Ref. 4.

Acknowledgments

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